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ON A METHOD OF REDUCING DUAL INTEGRAL EQUATIONS AND DUAL SERIES EQUATIONS TO INFINITE ALGEBRAIC SYSTEMS

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Mixed problems of the theory of elasticity, hydro- and aero-mechanics and of mathematical physics for the regions with partly infinite boundaries (a strip, layer, cylinder, wedge, cone, etc.), can often be reduced to studying dual integral equations. These equations usually involve the use of an integral transform generated by the Sturm-Liouville problem on a semi-infinite interval. Mixed problems for the finite regions (rectangle, circular plate, cylinder of finite length, sphere, etc.) can often be reduced to studying the dual series equations of some complete system of weight-orthonormed functions generated by the Sturm-Liouville problem on a finite interval. The present paper offers a method of reducing a wide class of such dual integral and series equations to infinite algebraic systems of special type. A way of investigating the infinite system obtained is indicated. The concept underlying the method was explained earlier by the author in [1].

1. Let a second order linear differential equation be given

$$(L - u^2)\, y = 0, \qquad Ly = r(x)\,[s(x)\, y']' + t(x)\, y \quad (a \leqslant x < \infty) \tag{1.1}$$

Here $s(x) > 0$ when $x \in (a, \infty)$ and $r(x)$ is a sign-definite function for $x \in (a, \infty)$. Let also the functions y and y' be bounded when $x \to \infty$ and

$$\alpha_2 y' + \alpha_1 y = 0 \tag{1.2}$$

when $x = a$.

By solving the Sturm-Liouville problem on a semi-infinite interval, we construct the following integral transform [2, 3]:

$$f(x) = \int_{-\infty}^{\infty} F(u)\, y(u, x)\, d\rho(u), \qquad F(u) = \int_{a}^{\infty} \frac{f(x)\, y(u, x)}{r(x)}\, dx \qquad (1.3)$$

where $y(u, x)$ is an eigenfunction and $\rho(u)$ is a nondecreasing spectral function.

Let us now consider the dual integral equation

$$\int_{-\infty}^{\infty} Q(u)\, K(u)\, y(u, x)\, d\rho(u) = f(x) \qquad (c \leqslant x \leqslant d) \qquad (1.4)$$

$$\int_{-\infty}^{\infty} Q(u)\, y(u, x)\, d\rho(u) = 0 \qquad (a \leqslant x < c,\ d < x < \infty)$$

$$K(u) = A\, \frac{P_1(u^2)}{P_2(u^2)} = A \prod_{n=1}^{\infty} \left(1 + \frac{u^2}{\delta_n^2}\right)\left(1 + \frac{u^2}{\gamma_n^2}\right)^{-1} \qquad (A = \mathrm{const}) \qquad (1.5)$$

Here $\pm i\delta_n$ and $\pm i\gamma_n$ denote the denumerable sets of simple zeros and poles. Let also δ_n and γ_n increase monotonously in modulo with increasing n thus ensuring the convergence of the infinite product (1.5), and let the following estimate hold on any correct system of contours C_n in the complex plane of the variable u:

$$K(u) \sim O(|u|^p), \quad p \leqslant 1, \ n \to \infty \qquad (1.6)$$

We shall restrict our investigation to the case

$$f(x) = y(i\varepsilon, x) \qquad (1.7)$$

remembering that in the general case the function $f(x)$ can be represented by the integral (1.3).

2. Making use of the fact that performing the operation L on the function $y(u, x)$ gives $u^2 y(u, x)$ and taking into account (1.5), we shall rewrite the first relation of the dual integral equation (1.4) as follows:

$$A(P_1(L)\, q(x) = P_2(L)\, y(i\varepsilon, x) = P_2(-\varepsilon^2)\, y(i\varepsilon, x) \qquad (c \leqslant x \leqslant d) \qquad (2.1)$$

$$q(x) = \int_{-\infty}^{\infty} Q(u)\, y(u, x)\, d\rho(u) \qquad (2.2)$$

Here $P_1(L)$ and $P_2(L)$ denote the differential operators in x of infinite order. The solution of the differential equation (2.1) in $q(x)$ can be written in the form:

$$q(x) = K^{-1}(i\varepsilon)\, y(i\varepsilon, x) + \sum_{n=1}^{\infty} H_n(x) \qquad (c \leqslant x \leqslant d) \qquad (2.3)$$

$$H_n(x) = C_n y(i\delta_n, x) + D_n \eta(i\delta_n, x) \qquad (2.4)$$

In (2.3) the first term is a particular solution of the inhomogeneous equation and it can be obtained by symbolic method, while the infinite sum gives the general solution of the homogeneous equation. In addition, $\eta(u, x)$ is a solution of (1.1) linearly independent of $y(u, x)$. Formula (2.3) and the second relation of (1.4) together define the function $q(x)$ for all $x \in [a, \infty)$ with the accuracy of up to the denumerable set of the constants C_n and D_n. Now, using the inversion formula (1.3), we can determine the unknown $Q(u)$ with the same accuracy. Taking into account the integral

$$\int_c^d \frac{y(v, x)\,\eta(w, x)}{r(x)}\,dx = \left\{\frac{s(x)}{v^2 - w^2}\left[y'(v, x)\,\eta(w, x) - y(v, x)\,\eta'(w, x)\right]\right\}_c^d \quad (2.5)$$

where $y(v, x)$ and $\eta(w, x)$ are any two solutions of (1.1) corresponding to $u = v$ and $u = w$, we obtain the following expression for $Q(u)$:

$$Q(u) = \left\{s(x)\left[-y(u, x)\left(\frac{y'(i\varepsilon, x)}{K(i\varepsilon)(\varepsilon^2 + u^2)} + \sum_{n=1}^{\infty}\frac{H_n'(x)}{\delta_n^2 + u^2}\right) + \right.\right. \quad (2.6)$$

$$\left.\left. y'(u, x)\left(\frac{y(i\varepsilon, x)}{K(i\varepsilon)(\varepsilon^2 + u^2)} + \sum_{n=1}^{\infty}\frac{H_n(x)}{\delta_n^2 + u^2}\right)\right]\right\}_c^d$$

3. The constants C_n and D_n must be determined from the condition that the solution (2.6) satisfies the first relation of the dual integral equation (1.4). Taking into account the representation of the function

$$f^*(x) = \begin{cases} y(i\varepsilon, x) & (c \leqslant x \leqslant d) \\ 0 & (a \leqslant x < c,\ d < x < \infty)\,, \end{cases} \quad (3.1)$$

in the form of the integral (1.3) and assuming that $K(i\delta_n) = 0$, we substitute (2.6) into (1.4) to obtain

$$s(d)\left\{K^{-1}(i\varepsilon)\left[y(i\varepsilon, d)\,S_d(\varepsilon, x) - y'(i\varepsilon, d)\,T_d(\varepsilon, x)\right] + \right. \quad (3.2)$$

$$\left.\sum_{n=1}^{\infty}\left[H_n(d)\,S_d(\delta_n, x) - H_n'(d)\,T_d(\delta_n, x)\right]\right\} -$$

$$s(c)\left\{K^{-1}(i\varepsilon)\left[y(i\varepsilon, c)\,S_c(\varepsilon, x) - y'(i\varepsilon, c)\,T_c(\varepsilon, x)\right] + \right.$$

$$\left.\sum_{n=1}^{\infty}\left[H_n(c)\,S_c(\delta_n, x) - H_n'(c)\,T_c(\delta_n, x)\right]\right\} = 0$$

where

$$T_r(\varkappa, x) = \int_{-\infty}^{\infty}\frac{K(u) - K(i\varkappa)}{\varkappa^2 + u^2}\,y(u, r)\,y(u, x)\,d\rho(u) \quad (3.3)$$

$$S_r(\varkappa, x) = \int_{-\infty}^{\infty}\frac{K(u) - K(i\varkappa)}{\varkappa^2 + u^2}\,y'(u, r)\,y(u, x)\,d\rho(u)$$

We note that by virtue of the assumptions made about the meromorphic function $K(u)$, the latter can be represented by the sum of principal values

$$K(u) = A - \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{g_m u^2}{\gamma_m(u^2 + \gamma_m^2)}\,, \quad g_m = \pi i\,\{[K^{-1}(i\gamma_m)]'\}^{-1} \quad (3.4)$$

Using (3.4) we can transform the expressions (3.3) as follows:

$$T_r(\varkappa, x) = -\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{g_m\gamma_m}{\gamma_m^2-\varkappa^2}\rho_r(\gamma_m, x) \tag{3.5}$$

$$S_r(\varkappa, x) = -\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{g_m\gamma_m}{\gamma_m^2-\varkappa^2}\sigma_r(\gamma_m, x)$$

$$\rho_r(\gamma_m, x) = \int_{-\infty}^{\infty}\frac{y(u, r)\,y(u, x)}{u^2+\gamma_m^2}\,d\rho(u) \tag{3.6}$$

$$\sigma_r(\gamma_m, x) = \int_{-\infty}^{\infty}\frac{y'(u, r)\,y(u, x)}{u^2+\gamma_m^2}\,d\rho(u)$$

4. Now we turn our attention to the fact that although the functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$ can increase with $x\to\infty$, it can easily be proved that a linear combination of these functions given by

$$\zeta(i\gamma_m, x) = \beta_1 y(i\gamma_m, x) + \beta_2\eta(i\gamma_m, x) \tag{4.1}$$

exists such that for $x\to\infty$

$$\zeta(i\gamma_m, x)\to 0, \qquad \zeta'(i\gamma_m, x)\to 0 \tag{4.2}$$

Writing the function $\zeta(i\gamma_m, x)$ in the form of the integral (1.3) and taking into account its properties (4.2), we obtain

$$\zeta(i\gamma_m, x) = -\chi(\zeta, a) \tag{4.3}$$

Here and in the following we have

$$\chi(\alpha, \beta) = s(\beta)\,[\alpha(i\gamma_m, \beta)\,\sigma_\beta(\gamma_m, x) - \alpha'(i\gamma_m, \beta)\,\rho_\beta(\gamma_m, x)]$$

Supplementing (4.3) with the relation

$$\alpha_1\rho_a(\gamma_m, x) + \alpha_2\sigma_a(\gamma_m, x) = 0 \tag{4.4}$$

which follows from the boundary condition (1.2), we obtain a set of two equations which yields $\rho_a(\gamma_m, x)$ and $\sigma_a(\gamma_m, x)$.

Let us now write the four discontinuous functions in the form of the integrals (1.3)

$$\left.\begin{matrix} y(i\gamma_m, x) & c\leqslant x\leqslant d) \\ 0 & (a\leqslant x< c,\ d< x<\infty)\end{matrix}\right\} = \chi(y, d) - \chi(y, c) \tag{4.5}$$

$$\left.\begin{matrix} \eta(i\gamma_m, x) & (c\leqslant x\leqslant d) \\ 0 & (a\leqslant x< c,\ d< x<\infty)\end{matrix}\right\} = \chi(\eta, d) - \chi(\eta, c)$$

$$\left.\begin{matrix} y(i\gamma_m, x) & (a\leqslant x\leqslant d) \\ 0 & (d< x<\infty)\end{matrix}\right\} = \chi(y, d) - \chi(y, a)$$

$$\left.\begin{matrix} \eta(i\gamma_m, x) & (a\leqslant x\leqslant d) \\ 0 & (d< x<\infty)\end{matrix}\right\} = \chi(\eta, d) - \chi(\eta, a)$$

Substituting into the last two relations of (4.5) the expressions obtained above for $\rho_a(\gamma_m, x)$ and $\sigma_a(\gamma_m, x)$, we obtain a system of four equations based on (4.5), and these equations yield

$$\begin{aligned}
\rho_d(\gamma_m, x) &= -[C(\gamma_m)\zeta^*(i\gamma_m, a)]^{-1}\zeta(i\gamma_m, d) M_m(x)\\
\rho_c(\gamma_m, x) &= -[C(\gamma_m)\zeta^*(i\gamma_m, a)]^{-1}\zeta(i\gamma_m, x) M_m(c)\\
\sigma_d(\gamma_m, x) &= -[C(\gamma_m)\zeta^*(i\gamma_m, a)]^{-1}\zeta'(i\gamma_m, d) M_m(x)\\
\sigma_c(\gamma_m, x) &= -[C(\gamma_m)\zeta^*(i\gamma_m, a)]^{-1}\zeta(i\gamma_m, x) M_m'(c)
\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}
\zeta^*(i\gamma_m, a) &= \alpha_1\zeta(i\gamma_m, a) + \alpha_2\zeta'(i\gamma_m, a)\\
M_m(x) &= y(i\gamma_m, x)\eta^*(i\gamma_m, a) - \eta(i\gamma_m, x) y^*(i\gamma_m, a)\\
y^*(i\gamma_m, a) &= \alpha_1 y(i\gamma_m, a) + \alpha_2 y'(i\gamma_m, a)\\
\eta^*(i\gamma_m, a) &= \alpha_1\eta(i\gamma_m, a) + \alpha_2\eta'(i\gamma_m, a)
\end{aligned} \tag{4.7}$$

We note that the functions $\rho_d(\gamma_m, x)$, $\rho_c(\gamma_m, x)$, $\sigma_d(\gamma_m, x)$ and $\sigma_c(\gamma_m, x)$ are all, by virtue of the formulas (4.1), (4.6) and (4.7), linear combinations of the functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$. It was also taken into account in (4.6) that the Wronskian of the functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$ is equal to $C(\gamma_m) s^{-1}(x)$.

Substituting now the expressions (4.6) and (3.5) into the relations (3.2) and equating to zero the sums of the coefficients accompanying like functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$, $m = 1, 2, \ldots$, we obtain two infinite algebraic systems for determining the constants C_n and D_n. One of these systems has the form

$$\begin{aligned}
&\frac{s(d)\eta^*(i\gamma_m, a)}{K(i\varepsilon)(\gamma_m^2-\varepsilon^2)}[y(i\varepsilon, d)\zeta'(i\gamma_m, d) - y'(i\varepsilon, d)\zeta(i\gamma_m, d)] -\\
&\frac{s(c)\beta_1}{K(i\varepsilon)(\gamma_m^2-\varepsilon^2)}[y(i\varepsilon, c)M_m'(c) - y'(i\varepsilon, c)M_m(c)] +\\
&\sum_{n=1}^{\infty}\frac{s(d)\eta^*(i\gamma_m, a)}{\gamma_m^2-\delta_n^2}[H_n(d)\zeta'(i\gamma_m, d) - H_n'(d)\zeta(i\gamma_m, d)] -\\
&\sum_{n=1}^{\infty}\frac{s(c)\beta_1}{\gamma_m^2-\delta_n^2}[H_n(c)M_m'(c) - H_n'(c)M_m(c)] = 0
\end{aligned} \tag{4.8}$$

and the other system is obtained from (4.8) by making the substitutions $\eta^* \to y^*$ and $\beta_1 \to \beta_2$.

5. Next we investigate the dual series equations of the form

$$\begin{aligned}
&\sum_{k=0}^{\infty} Q_k K(u_k) y(u_k, x) = f(x) \quad (c \leqslant x \leqslant d)\\
&\sum_{k=0}^{\infty} Q_k y(u_k, x) = 0 \quad (a \leqslant x < c,\ d < x \leqslant b)
\end{aligned} \tag{5.1}$$

Here $y(u_k, x)$ denote the system of the eigenfunctions of the Sturm-Liouville problem for the differential equation (1.1), with the boundary conditions

$$y'(u, a) + \alpha_1 y(u, a) = 0, \quad y'(u, b) + \alpha_2 y(u, b) = 0 \tag{5.2}$$

and u_k is a denumerable set of eigenvalues. We shall assume that in (1.1) the function

$s(x) > 0$ for $x \in (a, b)$ and $r(x)$ is a sign-definite function for $x \in (a, b)$. According to [2, 3], the functions $y(u_k, x)$ for $x \in [a, b]$, form a complete orthogonal system of weight $r^{-1}(x)$, and we shall normalize these functions with the same weight so that $\| y \| = 1$.

The function $K(u)$ in (5.1) has the form (1.5), and properties described in Sect. 1.

Below we shall limit ourselves to considering the case (1.7), remembering that in the general case the function $f(x)$ can be expanded into a series in $y(u_k, x)$.

Using the formulas (1.1) and (1.6), we write the first relation of (5.1) in the same form as in (2.1), where

$$q(x) = \sum_{k=0}^{\infty} Q_k y(u_k, x) \tag{5.3}$$

Here, as before, $P_1(L)$ and $P_2(L)$ are the differential operators in x of infinite order.

The solution of the differential equation (2.1) in $q(x)$ has the form (2.3) in which, as we indicated before, $\eta(u, x)$ is the solution of (1.1) linearly independent of $y(u, x)$.

The formula (2.3), together with the second relation of (5.1), define the function $q(x)$ for all $x \in [a, b]$ with the accuracy of up to the denumerable set of constants C_n and D_n. Using the orthogonal property of the functions $y(u_k, x)$, we can now find the unknown constants Q_k. Taking the integral (2.5) into account, we obtain the following expression for Q_k:

$$Q_k = Q(u_k) \tag{5.4}$$

where $Q(u)$ and $H_n(x)$ are given by the formulas (2.4) and (2.6).

6. The constants C_n and D_n must be determined from the condition that the solution (5.4) satisfies the first relation of (5.1). Taking into account the expansion of $f^*(x) = y(i\varepsilon, x), c \leqslant x \leqslant d, f^*(x) = 0, a \leqslant x < c, d < x \leqslant b$ into a series in $y(u_k, x)$, and the fact that $K(i\delta_n) = 0$, we obtain, after substituting (5.4) into (5.1), the relation (3.2) in which (3.3) are replaced by

$$T_r(\varkappa, x) = \sum_{k=0}^{\infty} \frac{K(u_k) - K(i\varkappa)}{u_k^2 + \varkappa^2} y(u_k, r) y(u_k, x) \tag{6.1}$$

$$S_r(\varkappa, x) = \sum_{k=0}^{\infty} \frac{K(u_k) - K(i\varkappa)}{u_k^2 + \varkappa^2} y'(u_k, r) y(u_k, x)$$

Using (3.4) we transform the expressions (6.1) into the form (3.5), where

$$\rho_r(\gamma_m, x) = \sum_{k=0}^{\infty} \frac{y(u_k, r) y(u_k, x)}{u_k^2 + \gamma_m^2}, \qquad \sigma_r(\gamma_m, x) = \sum_{k=0}^{\infty} \frac{y'(u_k, r) y(u_k, x)}{u_k^2 + \gamma_m^2} \tag{6.2}$$

We also have the following expansions:

$$y(i\gamma_m, x) = \chi(y, b) - \chi(y, a) \quad (a \leqslant x \leqslant b) \tag{6.3}$$

$$\left.\begin{matrix} y(i\gamma_m, x) & (a \leqslant x \leqslant d) \\ 0 & (d < x \leqslant b) \end{matrix}\right\} = \chi(y, d) - \chi(y, a)$$

$$\left.\begin{matrix} y(i\gamma_m, x) & (c \leqslant x \leqslant b) \\ 0 & (a \leqslant x < c) \end{matrix}\right\} = \chi(y, b) - \chi(y, c)$$

Supplementing (6.3) with the same expansions for the functions

$$\eta\ (i\gamma_m, x),\quad (a \leqslant x \leqslant b) \qquad \begin{cases}\eta\ (i\gamma_m, x)\ (a \leqslant x \leqslant d), \\ 0\ (d < x \leqslant b)\end{cases} \begin{cases}\eta\ (i\gamma_m, x) & (c \leqslant x \leqslant b) \\ 0\ (a \leqslant x < c)\end{cases} \tag{6.4}$$

and with the relations

$$\sigma_a(\gamma_m, x) + \alpha_1 \rho_a(\gamma_m, x) = 0, \qquad \sigma_b(\gamma_m, x) + \alpha_2\rho_b(\gamma_m, x) = 0 \tag{6.5}$$

which follow from the boundary conditions (5.2), we obtain a system of eight equations in $\rho_r(\gamma_m, x)$ and $\sigma_r(\gamma_m, x)$, $r = a, c, d, b$. Solving this system and taking into account the fact that the Wronskian of the functions $\eta(i\gamma_m, x)$ and $y(i\gamma_m, x)$ is equal to $C(\gamma_m)\, s^{-1}(x)$, we can write $\rho_r(\gamma_m, x)$ and $\sigma_r(\gamma_m, x)$ for $r = a, c, d, b$ in the form of the linear combinations of $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$. Substituting now into (3.2) the expressions (3.5) and (6.2) for the functions $T_r(\varkappa, x)$ and $S_r(\varkappa, x)$, as well as the linear expressions for $\rho_r(\gamma_m, x)$ and $\sigma_r(\gamma_m, x)$, $r = c, d$, and comparing in (3.2) the coefficients accompanying like functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$, $m = 1, 2, \ldots$, we obtain two infinite algebraic systems for determining the constants C_n and D_n

$$\begin{aligned}
&[K(i\varepsilon)(\gamma_m^2 - \varepsilon^2)]^{-1}[F_{12}(\gamma_m)\xi_d(\varepsilon, \gamma_m) - F_{21}(\gamma_m)\xi_c(\varepsilon, \gamma_m) + \\
&\quad F_{11}(\gamma_m)(\zeta_x(\gamma_m, \varepsilon))_c^{\,d}] + \sum_{n=1}^{\infty}(\gamma_m^2 - \delta_n^2)^{-1}\{C_n[F_{12}(\gamma_m)\xi_d(\delta_n, \gamma_m) - \\
&\quad F_{21}(\gamma_m)\xi_c(\delta_n, \gamma_m) + F_{11}(\gamma_m)(\zeta_x(\gamma_m, \delta_n))_c^{\,d}] + \\
&\quad D_n[F_{12}(\gamma_m)\zeta_d(\delta_n, \gamma_m) - F_{21}(\gamma_m)\zeta_c(\delta_n, \gamma_m) + \\
&\quad F_{11}(\gamma_m)(\vartheta_x(\gamma_m, \delta_n))_c^{\,d}]\} = 0
\end{aligned} \tag{6.6}$$

$$\begin{aligned}
&[K(i\varepsilon)(\gamma_m^2 - \varepsilon^2)]^{-1}[F_{22}(\gamma_m)(\xi_x(\varepsilon, \gamma_m))_c^{\,d} + F_{21}(\gamma_m)\zeta_d(\gamma_m, \varepsilon) - \\
&\quad F_{12}(\gamma_m)\zeta_c(\gamma_m, \varepsilon)] + \sum_{n=1}^{\infty}(\gamma_m^2 - \delta_n^2)^{-1}\{C_n[F_{22}(\gamma_m)(\xi_x(\delta_n, \gamma_m))_c^{\,d} + \\
&\quad F_{21}(\gamma_m)\zeta_d(\gamma_m, \delta_n) - F_{12}(\gamma_m)\quad \zeta_c(\gamma_m, \delta_n)] + \\
&\quad D_n[F_{22}(\gamma_m)(\zeta_x(\delta_n, \gamma_m))_c^{\,d} + F_{21}(\gamma_m)\vartheta_d(\gamma_m, \delta_n) - \\
&\quad F_{12}(\gamma_m)\vartheta_c(\gamma_m, \delta_n]\} = 0
\end{aligned}$$

where

$$\begin{aligned}
F_{kl}(\gamma) &= A_k(\gamma) B_l(\gamma) E^{-1}(\gamma) \\
E(\gamma) &= A_1(\gamma) B_2(\gamma) - A_2(\gamma) B_1(\gamma) \\
A_1(\gamma) &= y'(i\gamma, a) + \alpha_1 y(i\gamma, a), \quad A_2(\gamma) = \eta'(i\gamma, a) + \alpha_1\eta(i\gamma, a) \\
B_1(\gamma) &= y'(i\gamma, b) + \alpha_2 y(i\gamma, b), \quad B_2(\gamma) = \eta'(i\gamma, b) + \alpha_2\eta(i\gamma, b) \\
\xi_x(\mu, \gamma) &= s(x)\,[y(i\mu, x)\, y'(i\gamma, x) - y'(i\mu, x)\, y(i\gamma, x)] \\
\zeta_x(\mu, \gamma) &= s(x)\,[\eta(i\mu, x)\, y'(i\gamma, x) - \eta'(i\mu, x)\, y(i\gamma, x)] \\
\vartheta_x(\mu, \gamma) &= s(x)\,[\eta(i\mu, x)\, \eta'(i\gamma, x) - \eta'(i\mu, x)\, \eta(i\gamma, x)]
\end{aligned} \tag{6.7}$$

7. In deriving Eqs. (4.8) and (6.6) we have assumed the linear independence of the system of functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$, $m = 1, 2, \ldots$. This can be justified

[4] by recalling that for most real problems $|\gamma_m|$ and $|\delta_m| \sim m$ when $m \to \infty$. Using further the asymptotic formulas [2, 3, 5] for the functions $y(i\gamma_m, x)$ and $\eta(i\gamma_m, x)$ for large m we can confirm that in the cases which are interesting from the practical point of view, the infinite systems (4.8) and (6.6) can be regularized by constructing and inverting an infinite matrix with the elements $\tau_{mn} = (\gamma_m - \delta_n)^{-1}$.

In order to invert the above matrix exactly, let us consider the dual integral equation

$$\int_{-\infty}^{\infty} Q(u) K(u) e^{-iux} du = 2\pi e^{-\gamma_k x} \quad (0 \leqslant x < \infty) \tag{7.1}$$

$$\int_{-\infty}^{\infty} Q(u) e^{-iux} du = 0 \quad (-\infty < x < 0)$$

Applying to (7.1) the method expounded in Sects. 1 – 4 with small formal changes, we find

$$q(x) = \sum_{n=1}^{\infty} \sigma_{kn} e^{-\delta_n x} \quad (0 \leqslant x < \infty), \qquad Q(u) = \sum_{n=1}^{\infty} \sigma_{kn} (\delta_n - iu)^{-1} \tag{7.2}$$

where the constants σ_{kn} are obtained from the infinite system

$$\sum_{n=1}^{\infty} \sigma_{kn} \tau_{mn} = \pi g_m^{-1} \delta_{mk} \quad (m = 1, 2, \ldots) \tag{7.3}$$

Here the constants g_m have the form (3.4) and δ_{mk} is the Kronecker delta.

We now turn our attention to the fact that the dual equation (7.1) has a corresponding first order integral equation of the form

$$\int_0^{\infty} q(\xi) k(x - \xi) d\xi = e^{-\gamma_k x} \quad (0 \leqslant x < \infty), \tag{7.4}$$

$$k(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} K(u) e^{-iut} du$$

In [6] where Eqs. (7.4) and (1.5) were solved for the first time, the solution was sought in the form (7.2) and an infinite system (7.3) was obtained for determining the constants. On the other hand, an exact solution of (7.4) and (1.5) can be obtained using the Wiener-Hopf method [6, 7]. Comparing this exact solution with (7.2), we find, that the matrix σ_{kn}, which is an inverse of τ_{mn}, has the form

$$\sigma_{kn} = \{K_+'(-i\delta_n)[K_-^{-1}(i\gamma_k)]'(\gamma_k - \delta_n)\}^{-1} \tag{7.5}$$

$$K_+(u) = K_-(-u) = \sqrt{A} \prod_{m=1}^{\infty} \left(1 + \frac{u}{i\delta_m}\right)\left(1 + \frac{u}{i\gamma_m}\right)^{-1}$$

The method of solving an equation of the form (1.4) and (5.1) given above, was used by the author in the study of dual integral equations generated by the integral, sine and cosine Fourier transforms, and by the integral Hankel, Meler-Fock and Kontorovich-Lebedev transforms, and also to study the dual series equations in terms of the trigonometric, Bessel and associated Legendre functions. Examples of applications of the method to concrete problems are given in [8].

After making the appropriate generalizations, the method can be used to study the dual series equations in terms of the eigenfunctions of the Sturm-Liouville problem for a fourth order linear differential equation. Such dual equations are encountered when the method of homogeneous solutions is used to solve mixed problems.

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ON A METHOD OF SEPARATING THE STATE OF STRESS IN SHELLS OF NEGATIVE CURVATURE WITH ASYMPTOTIC EDGES

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The method of separating the state of stress is the following for shells with non-asymptotic edges: the total state of stress of the shell for which all the conditions of applicability of membrane theory are satisfied, is separated into the fundamental state of stress and simple edge effects. The boundary conditions are hence also separated: the tangential conditions are satisfied because of arbitrariness of the membrane theory, and the nontangential conditions because of the simple edge effects.

The possibility is shown in this paper of using this method to analyze shells of negative curvature with four asymptotic edges. The theory of the generalized edge effect has been constructed in [1]. Here, the formulas of the generalized edge effect are derived by another method for convenience in the subsequent exposition. Boundary conditions are formulated for membrane theory and the generalized edge effect for diverse edge fixings.

All the terminology, notation, equations and relations of shell theory are borrowed from [1].